Recent Developments in Neutron Star Thermal Evolution Theories and Observation(*)

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Abstract. Recent years have seen some significant progress in theoretical studies of physics of dense matter. Combined with the observational data now available from the successful launch of *Chandra* and *XMM/Newton* X-ray space missions as well as various lower-energy band observations, these developments now offer the hope for distinguishing various competing neutron star thermal evolution models. For instance, the latest theoretical and observational developments may already exclude both nucleon and kaon direct Urca cooling. In this way we can now have a realistic hope for determining various important properties, such as the composition, superfluidity, the equation of state and stellar radius. These developments should help us obtain deeper insight into the properties of dense matter.

INTRODUCTION

The launch of the *Einstein* Observatory gave the first hope for detecting thermal radiation directly from the surface of neutron stars (NSs). However, the temperatures obtained by the *Einstein* were only the upper limits[1]. *ROSAT* offered the first confirmed detections (not just upper limits) for such surface thermal radiation from at least three cooling neutron stars, PSR 0656+14, PSR 0630+18 (Geminga) and PSR 1055-52[2]. Recently the prospect for measuring the surface temperature of isolated NSs, as well as obtaining better upper limits, has increased significantly, thanks to the superior X-ray data from *Chandra* and *XMM/Newton*, as well as the data in the lower energy bands from optical-UV telescopes such as *Hubble Space Telescope*. Consequently, the number of possible surface temperature detections has already increased to at least seven[3]. Very recently *Chandra* offered an important upper limit to PSR J0205+6449 in 3C58[4]. At the same time, more careful and detailed theoretical investigation of various input microphysics has been in progress[5]-[9]. The current paper is meant as a progress report on these recent developments. Specifically, we try to demonstrate that distinguishing among various competing NS cooling theories has started to become possible, by careful comparison of improved theories with new observations[3][10][11].

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NEUTRON STAR COOLING THEORIES

The first detailed cooling calculations[12] showed that isolated NSs can be warm enough to be observable as X-ray sources for about a million years. After a supernova explosion a newly formed NS first cools via various neutrino emission mechanisms before the surface photon radiation takes over. Among the important factors which seriously affect the nature of NS cooling are: neutrino emission processes, superfluidity of constituent particles, composition, mass, and the equation of state (EOS)[3]. In this paper, for convenience, the conventional, slower neutrino cooling mechanisms, such as the modified Urca, plasmon neutrino and bremsstrahlung processes, will be called 'standard cooling'. On the other hand, the more 'exotic' extremely fast cooling processes, such as the direct Urca processes involving nucleons, hyperons, pions, kaons, and quarks, will be called 'nonstandard' processes[3].

The composition of NS interior is predominantly neutrons with only a small fraction of protons, electrons and muons when the interior density is not high (the central density $\rho^c <\sim 10^{15}$ gm/cm^3). For higher densities more 'exotic' particles, such as hyperons, pions, kaons and quarks, may dominate the central core. Therefore, when the star is less massive and hence less dense, we have a neutron star with the interior consisting predominantly of neutrons (no 'exotic' particles) and it will cool with the slower, 'standard' neutrino processes. On the other hand when ρ^c exceeds the transition density to the exotic matter ρ_{tr}, the transition from nucleons to 'exotic' particles takes place. Therefore, more massive stars, whose ρ^c exceeds ρ_{tr}, possess a central core consisting of the exotic particles. In that case, the nonstandard fast cooling takes over. Note, however, that if the proton fraction in the neutron matter is exceptionally high, i.e., $>\sim$ 15%, very fast cooling can take place in a NS without any exotic particles, through the nucleon direct Urca process. This can happen for a certain type of EOS models which allow such high proton concentration above a certain critical density[13]. In order to include this option, in the following discussion we will call any fast nonstandard process, an 'exotic process', rather than 'a process involving exotic particles'. The observational data suggest that there are at least two classes of NSs, the hotter and cooler. The most natural explanation is that the hotter stars are less massive and cool by slower cooling processes, while the cooler ones are more massive and cool by one of the fast nonstandard processes[3][10].

As the central collapsed star cools after a supernova explosion and the interior temperature falls below the superfluid critical temperature, T^{cr}, some constituent particles become superfluid. That causes suppression of both specific heat (and hence the internal energy) and all neutrino processes involving the superfluid particles. The net effect is that in the case of fast nonstandard cooling, the star cools more slowly and hence the surface temperature and luminosity will be higher at a given age during the neutrino cooling era, due to the suppression of neutrino emissivity. Therefore, nonstandard fast cooling will be no longer so fast if the superfluid energy gap, which is proportional to T^{cr}, is significant. In fact, if the gap is large enough, nonstandard cooling is fully suppressed and the cooling curve becomes essentially the same as the standard cooling curve. Therefore, depending on the size of the energy gap, a nonstandard cooling curve can lie anywhere between the standard curve and the nonstandard one without superfluid suppression.

In addition to various neutrino cooling mechanisms conventionally adopted in earlier

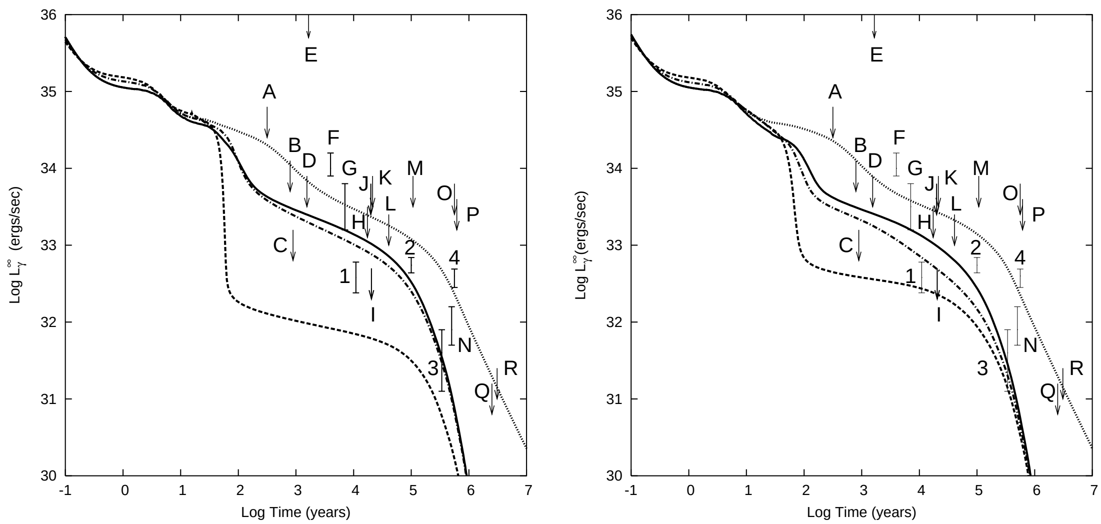

FIGURE 1. Thermal evolution curves with the newest observational data. In Fig. 1a (left panel) the dotted and solid curves refer to the standard cooling of M = 1.4$M_\odot$ neutron stars with and without heating, respectively, while the dot-dashed and dashed curves are for hyperon cooling of 1.6 and 1.8$M_\odot$ stars, respectively. In Fig. 1b (the right panel) the solid, dot-dashed and dashed curves refer to pion cooling of 1.4, 1.6 and 1.8$M_\odot$ stars, respectively. In the same figure the dotted curve refers to thermal evolution of a 1.4$M_\odot$ pion star with heating. The vertical bars refer to temperature detection data with error bars, while the downward arrows refer to the upper limits. The more accurate detection data are sown with numbers, for (1) the Vela pulsar, (2) PSR 0656+14, (3) Geminga, and (4) PSR 1055-52. The rest of the data shown are more rough estimates. Some of more interesting among these are shown with letters, as (A) Cas A point source, (B) the Crab pulsar, (C) PSR J0205+6449 in 3C58, (F) RX J0822-4300, (G) 1E1207.4-5209, (I) PSR 1046-58, (N) RX J1856-3754, and (R) PSR 1929+10. The complete list of all these data sources is found in T06a,b[10][11]. Sources RX J0002+62 and RX J0720.4-3125 are not shown because currently there are still too large uncertainties including the age estimate. See the text for further details.

calculations, recently the ‘Cooper pair neutrino emission’[14][15] was ‘rediscovered’ to be also important under certain circumstances. This process takes place when the participating particles become superfluid, and the net effect is to enhance, for some superfluid models, the neutrino emission right after the superfluidity sets in[15][16].

MOST RECENT THERMAL EVOLUTION MODELS

Most recently we calculated NS thermal evolution[2] adopting the most up-to-date microphysical input and a fully general relativistic, ‘exact’ evolutionary code (i.e., without making isothermal approximations). This code was originally constructed by Nomoto and Tsuruta (1987)[17] which has been continuously up-dated. Our input neutrino emissivity consists of all possible mechanisms, including Cooper pair emission. See Tsuruta et al. 2006a,b, hereafter referred to as T06a,b[10][11] for the details. In the models presented here, we consider thermal evolution of neutron stars which possess a central core

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[2] We adopt the expression ‘thermal evolution’ when we include not only cooling but also heating.

consisting of hyperons and pion condensates at high densities, which we conveniently call hyperon and pion stars. The results are summarized in Figure 1, where surface photon luminosity which corresponds to surface temperature (both to be observed at infinity), is shown as a function of age.

Fig. 1a (the left panel) shows thermal evolution of neutron (lower mass) and hyperon (higher mass) stars. The critical transition density from neutron to hyperon matter, ρ_{tr}^{Y}, is set at $4\rho_0$, which is estimated by nuclear theories[7]. (Here ρ_0 = 2.8 x 10^{14} gm/cm^3 is the nuclear density.) For $\rho < \rho_{tr}^{Y}$ we adopt the TNI6 EOS recently constructed for neutron matter by Takatsuka et al 2006[7], while for $\rho > \rho_{tr}^{Y}$ it becomes TNI6U, the same EOS but for hyperon matter[7]. This EOS is medium in stiffness[3] and it is very similar to the FP model adopted earlier, e.g. in Umeda et al. 1994[18] and Umeda, Tsuruta and Nomoto 1995[19]. As the superfluid model for hyperons we adopt the Ehime Model[7] and as the neutron superfluid model the OPEG-B Model[6], both recently constructed. The Cooper pair neutrino emissivity derived by Yakovlev, Levenfish, and Shibanov (1999)[14] is adopted for both neutrons and protons. For our heating calculations we adopt the vortex creep heating model with the heating parameter K = 10^{37} ergs m$^{-3/2}$ s^2, which is maximum in strength according to theoretical estimates[19][20], and magnetic field B = 10^{12} Gauss, reasonable for ordinary pulsars[21]. The other input parameters are the same as in Tsuruta 1998[21].

In Fig. 1a the dotted and solid curves refer to thermal evolution of a 1.4$M_\odot$ NS with and without heating, respectively. Since for these stars the central density $\rho^c < \rho_{tr}^{Y}$, they consist predominantly of neutrons and they cool by the slower 'standard' processes. The dot-dashed and dashed curves present cooling of 1.6$M_\odot$ and 1.8$M_\odot$ hyperon stars. For the TNI6U EOS adopted, we find that $\rho^c = \rho_{tr}^{Y}$ for a 1.5$M_\odot$ star. Therefore, our stars with mass larger than $\sim$ 1.5$M_\odot$ contain a hyperon core and hence the predominant cooling mechanism is the nonstandard hyperon direct Urca process. However, the 1.6$M_\odot$ star (dot-dashed) does not cool much faster than less massive neutron stars because the superfluid suppression is very large. The 1.8$M_\odot$ star (dashed) cools faster because the superfluid gap decreases significantly for this larger mass (and hence denser) star[4]. See T06a[10] for further details.

Fig. 1b (the right panel) shows thermal evolution of neutron stars with a pion core. The EOS adopted is 'TNI3P Model', which is a modified version of TNI3 EOS for neutron matter recently constructed[6]. This EOS is somewhat stiffer than medium. It was modified by T06b[11], to include pions for densities exceeding ρ_{tr}^{π}, the critical density for transition to pions. The pion transition density ρ_{tr}^{π} is set to be $2\rho_0$, significantly lower than that for hyperons, adopting the results of recent careful theoretical studies[8]. As the superfluid model for the pion-condensed phase we adopt the result from the most recent calculations by Tamagaki and Takatsuka (2006)[8] which indicates that the gaps for pion condensates are significantly larger for a significant range of densities above $2\rho_0$, although we assume that they decrease at higher densities. Other microphysical

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[3] Often an EOS is referred to as being 'stiff' when the consequent stellar model is more extended and less dense, while it is referred to as being 'soft' if it is more compact and denser for a given mass.

[4] Note that after the superfluidity sets in the gap first increases, reaches a peak and then decrease to zero as density increases.

input is the same as in Fig. 1a.

In Fig. 1b the dotted and solid curves refer to thermal evolution of $1.4M_\odot$ pion stars with and without heating, respectively. Since the transition density is low for pions we find that the central density of a $1.4M_\odot$ star already exceeds the pion transition density even for this relatively stiff EOS chosen, and hence its core already consists of pion condensates. However, for these stars the gap, and hence T^{cr}, is so large that the superfluid suppression is essentially complete, which means the curves lie essentially in the same positions as the standard cooling. The dot-dashed and dashed curves present cooling of $1.6M_\odot$ and $1.8M_\odot$ pion stars, respectively. For the higher central density of these more massive stars the superfluid gap decreases and hence these stars cool faster. However, for these stars the gap is still large enough to keep the superfluid suppression significant. See T06b for further details.

COMPARISON WITH OBSERVATION

In Fig. 1 thermal evolution curves are compared with the latest observational data. We may note that the data suggest the existence of at least two classes of sources, hotter stars (e.g., (F) RX J0822-4300, (G) 1E1207.4-5209, (2) PSR 0656+14, (N) J1856-4754, and (4) PSR 1055-52), and cooler stars (e.g., (C) PSR J0205+6449 in 3C58, (1) the Vela pulsar and (I) PSR 1046-58). The hotter sources are consistent with thermal evolution of less massive stars such as $1.4M_\odot$ stars. For PSR 1055-52(4) the age uncertainty is relatively large, but still it will probably require at least moderate heating. Source (F) is slightly above the dotted curves, but for this source the distance is quite uncertain and if it is closer that will bring the data point down. Also if the star has magnetic envelopes with light elements such as Hydrogen cooling will be slower and that will bring the curves somewhat higher[22]. Note that for this source the best fit to the spectral data requires a Hydrogen atmosphere[10].

Comparison of cooler star data with pion and hyperon curves confirms the earlier conclusion[3][21][23] that nonstandard cooling of more massive stars is required for these cooler data. In this case we find that significant superfluid suppression is required, at least for (1) the Vela pulsar detection data. The age uncertainty should not affect this conclusion, especially for younger cooler sources such as the pulsars in (C) 3C58 and (1) Vela, because the slope of the curves in these younger years is relatively flat. Also the uncertainty for the age estimate from pulsar spins is small for younger pulsars, at most a factor of 2 or so. Note that for the pulsar in 3C58 the difference between the age estimates coming from the SNR data and pulsar data is very small[10].

Recently a possibility for very cold NSs in at least four supernova remnants (SNRs) was reported by Kaplan et al. 2004[24]. These authors note that if there is a NS in these SNRs the upper limits to their surface luminosity should be very low[5]. We do not place these upper limits in our Fig. 1 because their data are given as L_x(0.5 - 10 keV), the X-ray luminosity within the limited window between 0.5 - 2 Kev. That should be significantly

[5] these authors pointed out that although some of these SNRs may contain no compact collapsed objects or the compact remnants may be black holes, it is quite unlikely that none of them contains a NS.

lower than L_{bol}, the total bolometric luminosity over all wavelengths, which should be the one to be compared with theoretical luminosity in the cooling curves. For instance, $L_{bol} \sim 80\ L_x$(0.5 - 10 keV) for PSR 0656+14 (the former, $\sim 8 \times 10^{32}$ ergs/s, vs the latter, $\sim 10^{31}$ ergs/s). However, even so, we note that these upper limits are safely below the standard cooling curves, and hence a nonstandard fast cooling scenario is required.

DISCUSSION

Until recently it was thought that at least for binary pulsars observations offered stringent constraints on the mass of a NS, to be very close to $1.4M_\odot$[21]. If this evidence extends to isolated NSs also, then the EOS should be severely constrained because it has to be such that the mass of the star whose central density is very close to the transition density (where the nonstandard process sets in) should be very close to $1.4M_\odot$[21]. Very recent observational data, however, suggest that the mass range should be much broader, ~ 1 – $2M_\odot$[9][25]. If so, that still should give some useful constraint on the EOS. For instance, a very soft EOS, such as the BPS Model[21], should be excluded because for this EOS the maximum mass is only $\sim 1.5M_\odot$, and hence stars with mass larger than this cannot be explained. That is why we chose medium to stiff EOSs for our models. We chose a stiffer EOS for pion stars because the transition to pions takes place at lower densities.

The qualitative behavior of all nonstandard scenarios is similar if their transition density is the same[21]. However, here we try to demonstrate that it is still possible to offer comprehensive assessment of at least which options are more likely while which are less likely. First of all, we note that all of the nonstandard mechanisms without suitable suppression are too fast for all the detection data. Significant suppression of neutrino emissivity due to superfluidity is required, to be consistent with cooler stars such as the Vela pulsar. However, Takatsuka and Tamagaki (1997), hereafter TT97[26] already showed, through careful microphysical calculations, that for neutron matter with such high proton concentration as to permit the nucleon direct Urca, the superfluid critical temperature T^{cr} should be extremely low, $\sim$ several x 10^7 K, not only for neutrons but also for protons. Here we emphasize that this conclusion does not depend on the nuclear models adopted for the calculations. On the other hand, most of the the observed NSs, which are to be compared with cooling curves, are hotter (the core temperature being typically $\sim 10^8$ K to several times 10^8 K). That means *the core particles are not yet in the superfluid state* in these observed NSs. Conclusion is that *a star cooling with nucleon direct Urca would be too cold* to be consistent with these detection data. The same argument applies to kaon cooling also[27]. Further details are found in [3][10][11][23].

As to the hyperon cooling scinario, we find that that will be a viable option if recently constructed hyperon superfluid gap models (which include the Ehime Model adopted here)[7] are valid. Recently the Gifu-Kyoto nuclear experimental group[28] reported that the superfluid gap for hyperons would be much smaller. If so, hyperon cooling also would be in trouble. However, since then their experiment has not been confirmed by follow-up experiments. We find that the pion cooling option is still valid.

A few other groups have calculated neutron star cooling. Due to lack of space here

we comment on only the recent major work by Yakovlev et al 2004, hereafter referred to as Y04[16]. Although these authors sometimes adopted simplified ‘toy models’ with the isothermal and other various approximations, their results and ours generally agree, at least qualitatively, when similar input is applied. There are, however, some serious differences. For instance,

(i) In an effort to bring up the standard cooling curves to explain a hot pulsar PSR 1055, these authors conclude that neutron superfluidity must be so weak as to be negligible. However, this conclusion contradicts with the results of serious theoretical studies of neutron superfluidity, which find that neutron superfluidity could not be so small for normal neutron matter where proton concentration is small[6]. On the other hand, we have shown (see Fig. 1) that this apparent discrepancy disappears even with models with significant neutron superfluidity when heating is included. Also, it may be pointed out that the age uncertainty is rather large for this pulsar since it is older[10]. Therefore, the unrealistic assumption of negligible neutron superfluidity for normal neutron matter is not required.

(ii) To explain cooler stars, these authors chose the nucleon direct Urca process, which they called Durca, as their nonstandard cooling option. Also, to explain both hot PSR 1055 and cooler Vela pulsar Y04 require their models to possess large proton superfluidity and yet negligible neutron superfluidity. However, TT97[26] already showed that that is impossible for models where the Durca option works. Specifically, for Durca to operate proton concentration must be significant. In such a case TT97 showed that both neutron and proton superfluidity must be very small - it cannot be that one is very high while the other negligibly small. In other words the models by Y04 are unphysical. Also, for such models where Durca can operate it is impossible for proton superfluidity to be strong enough to offer sufficient suppression to explain the Vela pulsar. Here we emphasize that their models are based on studies of normal neutron matter where proton concentration is small, while that argument breaks down for special models with high proton concentration which will allow nucleon Durca to operate.

SUMMARY AND CONCLUDING REMARKS

We have shown that the most up-to-date observed temperature data are consistent with the current thermal evolution theories of isolated NSs if less massive stars are warmer while more massive stars are cooler. The comparison of theory with observation, especially with the low temperature upper limit for the pulsar in 3C58, shows that fast nonstandard cooling is required for cooler stars. The need for nonstandard cooling is further strengthened by the recent report by Kaplan et al.[24] for the very low upper limits to neutron stars possibly present in some of four SNRs.

Among various nonstandard cooling scenarios, both nucleon Durca and kaon cooling may be excluded. The major reason is that for nucleon Durca to be operative, high proton concentration is required, which weakens superfluidity of both protons and neutrons. Then nucleon Durca will be too fast to be consistent with e.g., the Vela pulsar data. Similar argument applies to kaon cooling. Hyperon cooling may be in trouble if the hyperon superfluid gap should be so small as reported by recent nuclear experiments,

although this report is yet to be confirmed by follow-up experiments. On the other hand, pion cooling is still consistent with both observation and theory. The conclusion is that *the presence of 'exotic' particles, possibly pion condensates, will be required within a very dense star.* If the need for larger mass stars most recently reported for binary pulsars is confirmed, the very soft EOSs should be excluded.

The capability of constraining the composition of NS interior matter purely through observation alone will be limited, due to often very large uncertainties, mainly for stellar distance and age. Therefore, it will be very important to *exhaust all theoretical resources*. Theoretical uncertainties are also very large, especially in the supranuclear density regime. However, here we emphasize that we should still be able to set *acceptable ranges* of theoretical feasibility, at least to separate models more-likely from those less-likely.

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